

Optimal Transport: an application to the RADAR Recognition Process for deinterleaving RADAR pulses and identifying emitters

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AVANTIX

Supervision & funding

- **Industrial thesis**: Artificial Intelligence for Passive RADAR
 - The Signals and System Laboratory (L2S) from CentraleSupélec at Paris-Saclay University
 - Co-supervisor: Gilles Chardon
 - Thesis director: Frédéric Pascal
 - Avantix, a French company specialized in Electronic Warfare:
 - Co-supervisor: Chakib Belafdil
 - Supervisor: Jean-Daniel Busi



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- 1 Introduction
- 2 Data, modeling, and simulation of radar signals
- 3 Deinterleaving a RADAR signal
- 4 Identification of emitter
- 5 Performance evaluation
- 6 Conclusion

The challenges of RADAR signal processing

- Avantix designs critical high-tech systems to provide homeland protection and military forces.
- Provides strategic information to military platforms:
 - Identifying the enemies gives a considerable advantage (decision-making, commitment ...),
 - Anticipating the actions and setting up adapted measures.
- Use and development of machine learning tools for passive RADAR.
- Improving the RADAR Recognition Process ^[1, 2] with Optimal Transport theory ^[3, 4, 5]:
 - ① Deinterleaving RADAR pulses from a signal,
 - ② Identification of RADAR from a set of pulses.

RAdio Detection And Ranging

What is a passive RADAR?

- Emit no electromagnetic waves
- Only receivers
- Detect, track, and identify other emitters(s)
 - Monitoring discreetly
 - Minimizing interference or disruption
 - Cost optimization

➡ Focus on identification for ELINT [1, 6, 7]



Figure: ELINT RESM.

RADAR Recognition Process

① Deinterleaving RADAR pulses from a signal

👉 **Definition:** Separating and grouping the mixed pulses coming from an unknown number of emitters in a signal.

? Existing methods:

- Pulse repetition intervals, frequency, direction of arrival [8, 9, 6],
- Clustering [10, 11, 12],
- Deep Learning [13, 14, 15],
- ...

✗ **Limitations:** Supervised methods, only efficient on simple RADAR, need a lot of data...

👉 **Our solution:** Pulses separation with HDBSCAN and cluster fusion with Hierarchical Agglomerative Clustering using Optimal Transport Distances.

RADAR Recognition Process

② Identification of RADAR from a set of pulses

👉 **Definition:** Feature extraction and comparison with a reference database to identify the RADAR present in a signal.

? Existing methods:

- Time-frequency analysis [16, 17],
- Deep learning [18, 19, 20],
- ...

✗ **Limitations:** Supervised methods, few classes, model retrained, performance depends on simulator accuracy, lots of settings ...

👉 **Our solution:** Developing a distance between a set of pulses and emitter classes using the Optimal Transport Distances and the missing pulses rate modelization for the Pulse Repetition of Intervals.

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Methodology

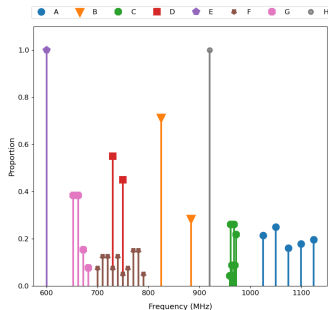
- Methods developed from a simulator and validated on real data:
 - Simulator developed by our knowledge of the radar environment,
 - Receptor static, omnidirectional, and has known detection threshold,
 - No consideration of adverse measures introducing disruptive signals.
- 2 steps for signal simulation:
 - ① Creation of a database gathering the emitters' characteristics,
 - ② Simulation of a signal from these characteristics according to the parameters set by the user (noise, number of emitters in the signal, size...).

Emitters class database

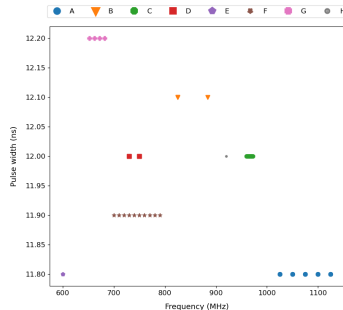
Emitter described by:

- **Technical emitter parameters:** Power Transmitted (Pow_t), transmit antenna gain (Gn_t), scan period (S), or antenna length (y).
- **Pulses parameters:** Frequency (F , f_n), pulse width (PW, pw_n), level (G , g_n), or pulse repetition interval pattern (PRI, pri_n).

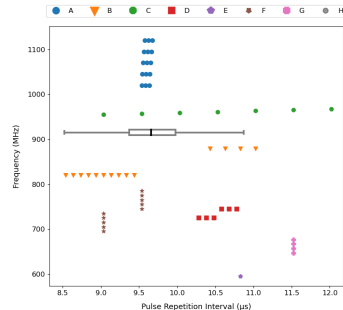
Emitters characteristics example



Frequency.



Frequency x Pulse Width.



Pulse Repetition Interval x Frequency.

Figure: Emitters characteristics representation.

Pulse Repetition of Interval representation

- Discret process:

$$\mu_j = \sum_{n=1}^N \alpha_n \delta_{pri_n}, \quad (1)$$

with j the emitter index, N the number of PRIs on which the emitter transmits, α the proportion of the PRI, and δ , the Dirac mass (with $\sum \alpha_n = 1$).

- Random process:

$$P(x)_j = \frac{1}{\sigma_j \sqrt{2\pi}} \exp\left(-\frac{(x - \mu_j)^2}{2\sigma_j^2}\right), \quad (2)$$

with j the emitter index, μ of the PRI average value, and σ its standard deviation.

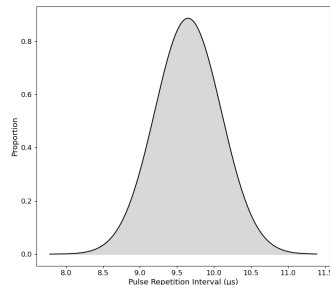


Figure: Random Pulse Repetition Interval.

Data acquisition

- Each train is composed of pulses, described by only 4 features:

$$pdw_i = \begin{pmatrix} f_1 & pw_1 & g_1 & toa_1 \\ f_2 & pw_2 & g_2 & toa_2 \\ \vdots & \vdots & \vdots & \vdots \\ f_m & pw_m & g_m & toa_m \end{pmatrix}, \quad (3)$$

with i the train, m the pulses index, f the frequency, pw the pulse width, g the level and toa the time of arrival.

- Computation of the difference of time of arrival (DTOA):
 $dtoa_m = toa_m - toa_{m-1}$.

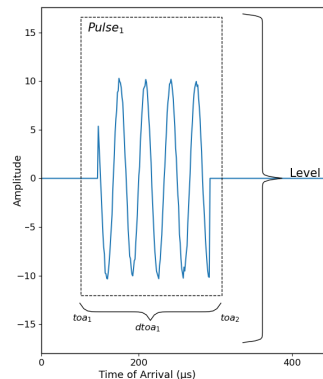
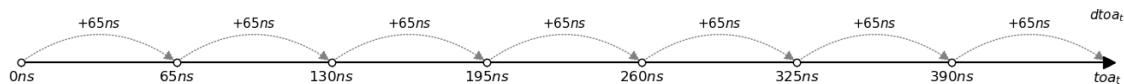


Figure: Example of a pulse from an intercepted signal.

Relation between the DTOA and the PRI

- Example of an Emitter with one PRI of $65ns$:



No noise, single emitter and all pulses intercepted $\Rightarrow PRI \equiv DTOA$

RADAR Signal example

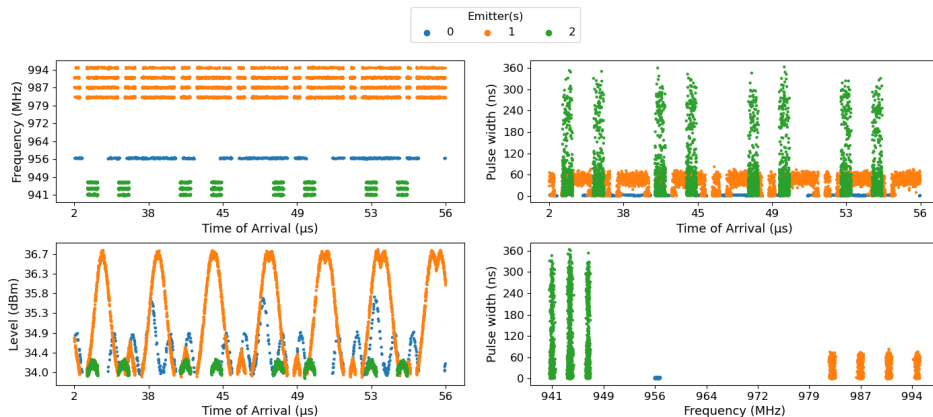
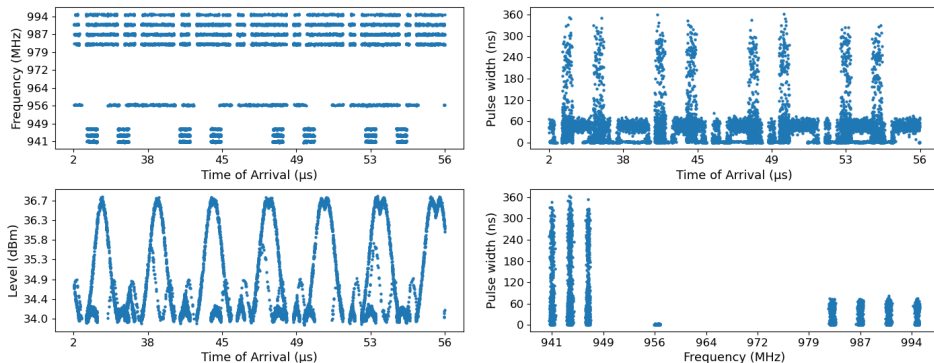


Figure: Example of a **simple** simulated signal grouping 8917 pulses from three emitters identified by color.

Application of the **RRP** on an intercepted signal



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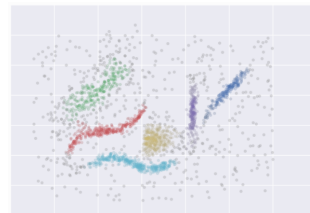
Hierarchical Agglomerative Clustering using Optimal Transport to deinterleave pulses - **HACOT**

2-step strategy

- ① **Pulse separation**: Apply HDBSCAN [21] from frequency (F) and pulse width (PW).
 - ② **Cluster aggregation**: Apply a Hierarchical Agglomerative Clustering based on Optimal Transport distances [3, 22] from time of arrival (TOA) and level (G).
-

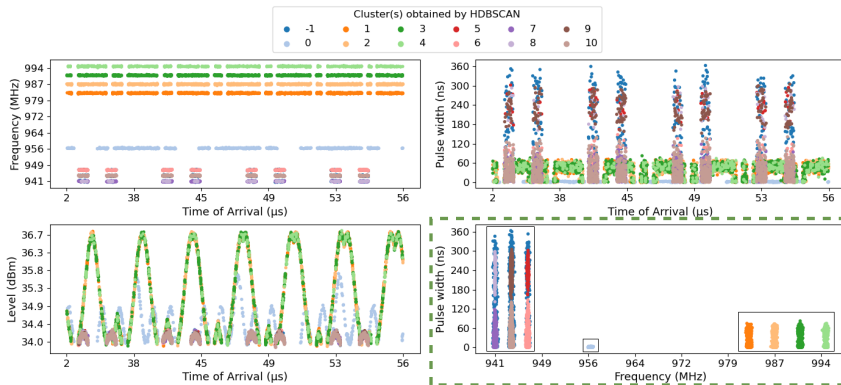
Step 1: Pulses separation with HDBSCAN

- Application of HDBSCAN on frequency (F) and pulse width (PW).
- Why HDBSCAN?
 - Unsupervised algorithm,
 - Search clusters of varying densities,
 - Few parameters to optimize,
 - May account for outliers (bring robustness to the proposed methodology).
- Over-estimation of the number of clusters:
 - Some emitters don't transmit continuously,
 - Each cluster must not contain the pulses of different emitters.



Credit: hdbscan.readthedocs.io.

HBSCAN outputs



Clustering plane

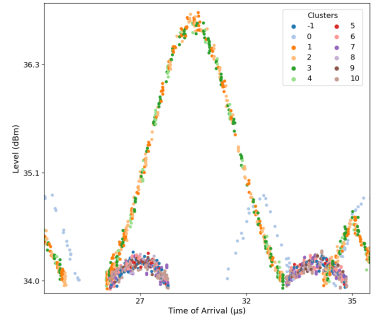
Figure: Clustering results performed from frequency and pulse width. Each color represents clusters and a class of outliers (-1).

→ How to group these clusters?

Step 2: Cluster aggregation with a Hierarchical Agglomerative Clustering based on Optimal Transport distances

Methodology

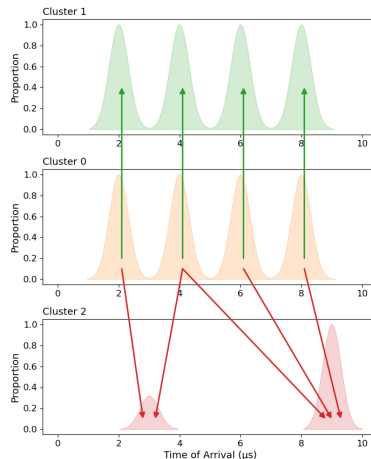
- I **Cluster representation** by a measure from time of arrival (TOA) and level (G),
- II **Distances computation** between clusters using Optimal Transport distances,
- III **Aggregation** of the two closest clusters,
- IV **Updating distances**,
- V **Repeat** until a single cluster is obtained.



➔ How to define a distance between clusters using optimal transport distances?

Optimal Transport approach in 1 dimension

- **Intuition:** Compare clusters distributions to define a distance.
- Cluster 0 and Cluster 1 belong to the same emitter:
 - ✓ Similar distribution,
 - ✓ Low transport cost.
- Cluster 0 and Cluster 2 don't belong to the same emitter:
 - ✗ Different distribution,
 - ✗ High transport cost.



Optimal Transport (OT) explanation

- Consider transporting two discrete distributions $\mu_s = \sum_{n=1}^{N_s} \alpha_n \delta_{x_n}$ toward $\mu_t = \sum_{m=1}^{N_t} \beta_m \delta_{y_m}$.
- The total cost of a transport plan \mathbf{P} is:

$$C(\mathbf{P}) = \sum_{n=1}^{N_s} \sum_{m=1}^{N_t} c(x_n, y_m) \mathbf{P}(x_n, y_m) = \langle \mathbf{C}, \mathbf{P} \rangle, \quad (4)$$

with $C(\mathbf{P})$ the total cost, $c(x_n, y_m)$ the cost of transporting unit mass from x_n to y_m (transport cost) and $\mathbf{P}(x_n, y_m)$ the quantity of mass taken from x_n to y_m (transport plan).

- The optimal transport plan is a simple minimization problem:

$$\mathbf{P}^* = \underset{\mathbf{P} \in (\mathbb{R}^+)^{N_s \times N_t}}{\operatorname{argmin}} \langle \mathbf{C}, \mathbf{P} \rangle \text{ such that } \mathbf{P} \mathbf{1}_{N_t} = \alpha \text{ and } \mathbf{P}^T \mathbf{1}_{N_s} = \beta. \quad (5)$$

Decisional model for pruning dendrogram based on unsupervised metrics and Optimal Transport distances

- Set aside Cluster 8,
- Decision model for pruning dendrogram based on:
 - Gap Score to the optimal transport distances,
 - Silhouette score [23],
 - Calinski-Harabasz score [24],
 - Davies-Bouldin score [25].

➡ **Majority rule** to get the optimal threshold.

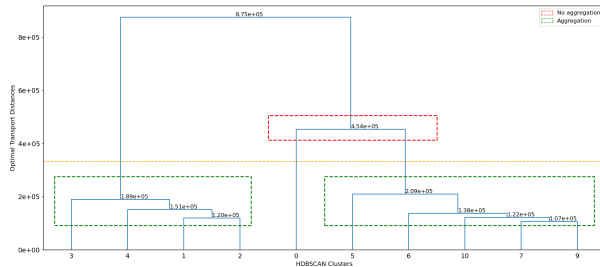


Figure: Cluster aggregations with Optimal Transport distances, the orange line highlights the optimal cut.

Hierarchical approach outputs

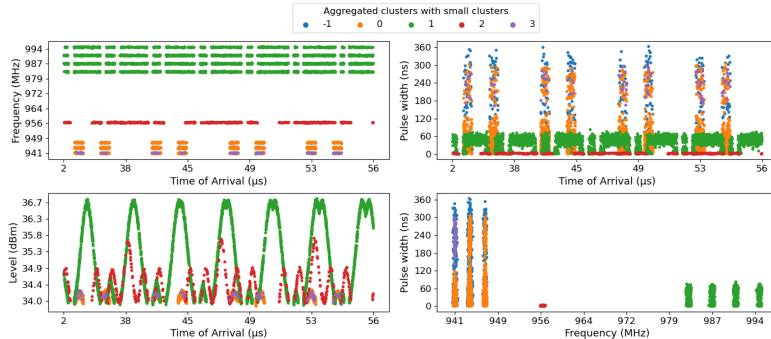


Figure: Aggregated clusters obtained according to the decision model and dendrogram resulting from the Hierarchical Agglomerative Clustering using Optimal Transport distances. Each color identifies an aggregated cluster with an outlier class (-1).

Dealing with excluded cluster

Application of a non-parametric method:

- 1 Estimation of the probability density of time of arrival of the extensive clusters i using a kernel density estimator with Gaussian kernel [26, 27, 28]:

$$\hat{f}_i(X) = \frac{1}{Nh} \sum_{i=1}^N K\left(\frac{x - x_i}{h}\right), \quad (6)$$

with N , the number of pulses, K , the kernel, h , and the smoothing parameter.

- 2 Association of an excluded cluster to an extensive cluster by maximum likelihood estimation:

$$L_i = \prod_{i=1}^N \hat{f}_i(t_n), \quad (7)$$

$$i^* = \operatorname{argmax} L_i. \quad (8)$$

Final grouping

- Perfect grouping
- Statistics to evaluate the groupings:
 - ✓ Homogeneity [29] = 1
 - ✓ Completeness [29] = 1
 - ⚠ Unclassified rate = 4.5%

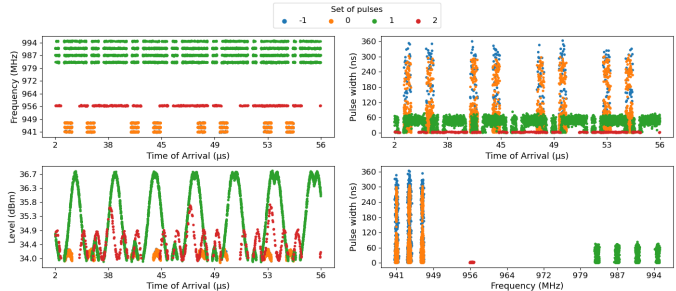
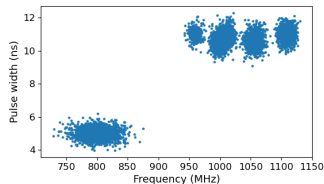
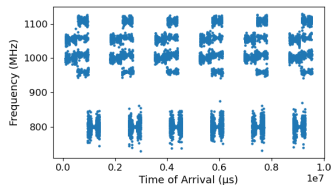
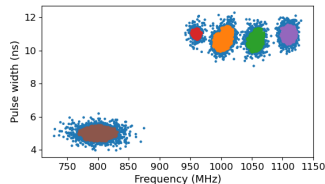
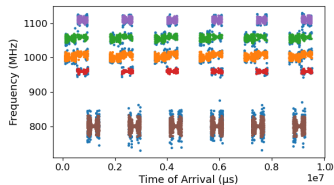


Figure: Final grouping identifying by a color with an outlier class (-1).

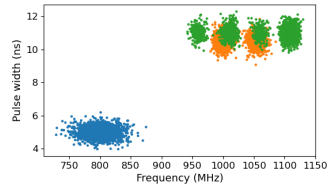
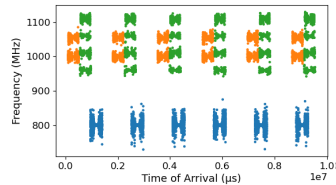
But sometimes...the reality is more complex...



Intercepted signal.



HDBSCAN outputs made in 2D.



Ground labels.

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Improved Hierarchical Agglomerative Clustering using Optimal Transport to deinterleave pulses - **IHACOT**

3-step strategy

- 1 **Pulse separation**: Apply HDBSCAN from time of arrival (TOA), frequency (F), and pulse width (PW).
 - 2 **Pre-clusters aggregation**: Apply a hierarchical agglomerative clustering from frequency (F) based on cluster averages pulse frequencies (\bar{f}_n) and width (\overline{pw}_n).
 - 3 **Cluster aggregation**: Apply hierarchical agglomerative clustering based on optimal transport distances ^[3, 22] from time of arrival (TOA) and level (G).
-

Data description

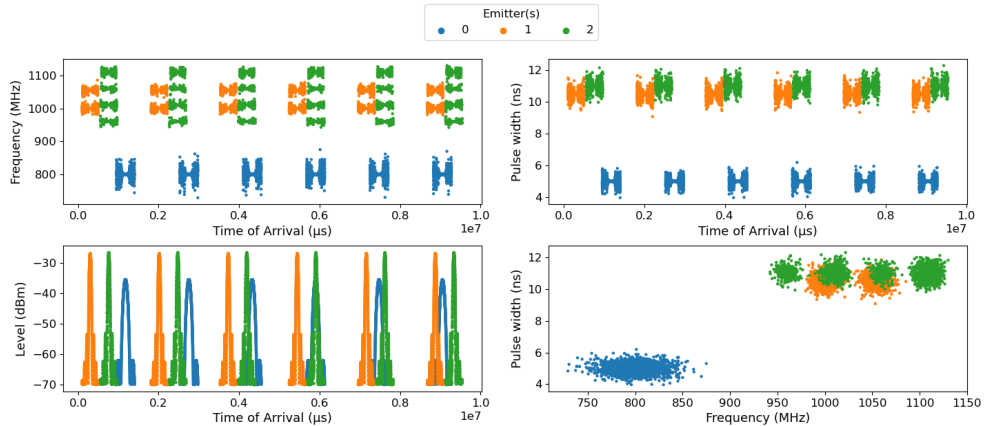


Figure: Example of a simulated signal gathering 22760 pulses of three emitters, represented by a color.

Step 1: Pulses separation with HDBSCAN

- Application of HDBSCAN on time of arrival, frequency, and pulse width:

- Better separability,
- Avoid pulses mixing,
- Temporal clusters.

→ How to group simultaneously active clusters characterized by the same frequencies and pulse widths?

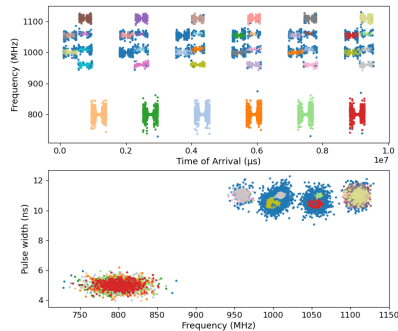


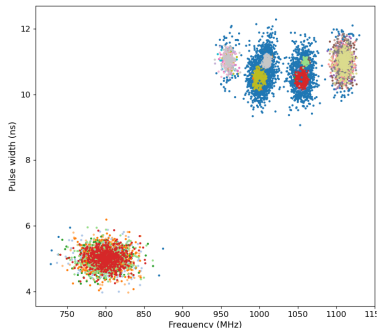
Figure: Clustering results performed in 3 dimensions. HDBSCAN detects 42 clusters with an outliers class (-1) identified by colors.

Step 2: Pre-clusters aggregation with a Hierarchical Agglomerative Clustering based on Euclidean distance

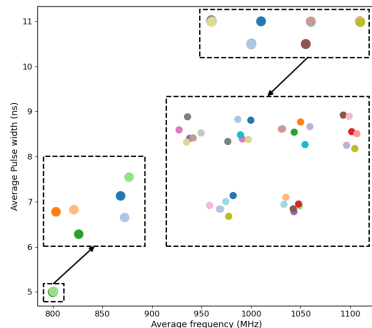
Methodology

- ① **Cluster separation** by estimating cluster average pulse frequencies ($\overline{f_n}$) and width ($\overline{pw_n}$),
 - ② **Cluster aggregation** by applying a Hierarchical Agglomerative Clustering using Euclidean distances from frequency (F),
 - ③ **Dendrogram pruning** according to a decisional model from time of arrival (TOA).
-

Step 2.1: Estimation of clusters averages frequencies and pulse widths



Clusters representation in the (f_n, pw_n) plane.



Cluster averages' representation in the (f_n, pw_n) plane.

Figure: HDBSCAN clustering results performed in 3 dimensions. HDBSCAN detects 42 clusters with an outliers class identified by colors.

Step 2.2: Cluster aggregation with Hierarchical Agglomerative Clustering

- Apply a Hierarchical Agglomerative Clustering based on Euclidean distance from frequency,
- Set aside pulse width due to excessive estimation error,
- Previous decisional model is not suitable due to many clusters.

→ How to stop aggregation?

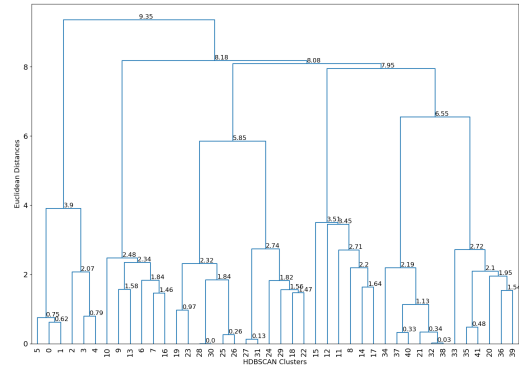


Figure: Dendrogram representing the aggregations at each iteration with the logarithm Euclidean distances values displayed.

Step 2.3: Improved decisional model for pruning the dendrogram based on statistical test

Methodology

- I **Selection of the first two aggregated clusters** according to the hierarchical structure,
 - II **Computation of the Kolmogorov-Smirnov test** from time of arrival (TOA),
 - III **Comparison of the p -value** with a **confidence threshold α** :
 - if $p\text{-value} < \alpha$: do not consider all aggregation using these clusters,
 - else select the next aggregation composed of the two aggregated clusters and repeat from (II).
 - IV **Repeat** until all aggregations are evaluated.
-

➔ How to set the confidence threshold?

The Elbow Trick for fixing the confidence level

- Sorting p -values in ascending order,
- Identifying the first breakpoint,
- Confidence level:
 $\alpha_{ks} = 4.74 \times 10^{-4}$.

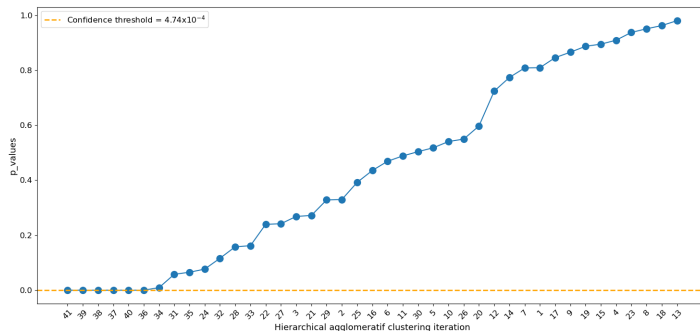


Figure: p -values sorted according to the Kolmogorov-Smirnov test, and the orange line highlights the estimated confidence level.

Elbow Tricks outputs

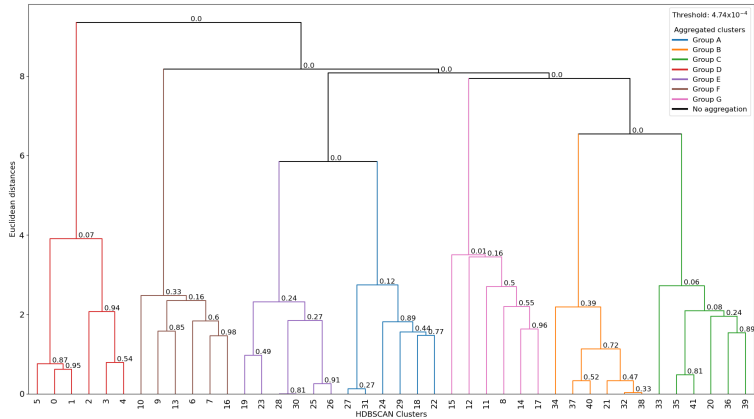


Figure: Hierarchical agglomerative clustering results according to the Kolmogorov-Smirnov test outputs. Each color identifies a cluster aggregate, and the Kolmogorov-Smirnov p -values are displayed.

Pre-clusters aggregation outputs

- Require another grouping phase,
- Application of the previous Hierarchical Agglomerative Clustering based on Optimal Transport distances [3, 22] from time of arrival (TOA) and level (G).

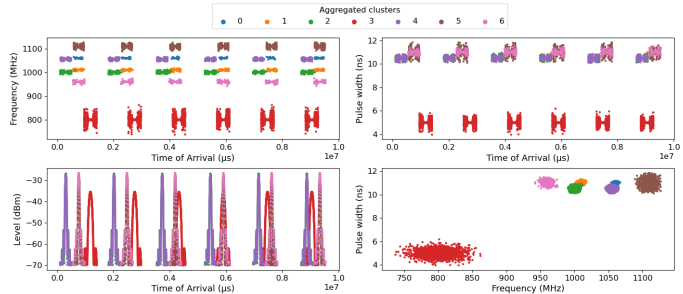


Figure: Aggregated clusters obtained following the Hierarchical Agglomerative Clustering using Euclidean distances pruning with Kolmogorov-smirnov test. Each color represents an aggregated cluster.

Step 3: Cluster aggregation with a Hierarchical Agglomerative Clustering based on Optimal Transport distance

- Satisfactory separation
- Statistics to evaluate the groupings:
 - ✓ Homogeneity [29] = 1
 - ✓ Completeness [29] = 1
 - ⚠ Unclassified rate = 9%

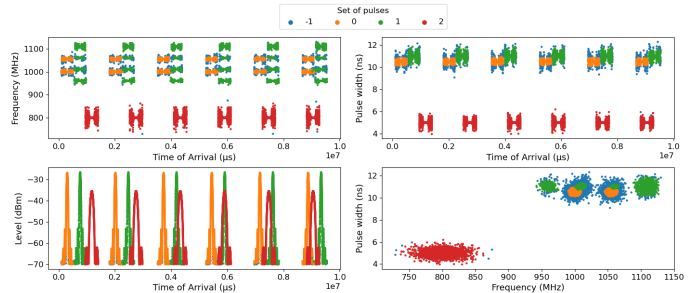


Figure: Final grouping after applied Hierarchical Agglomerative Clustering combined with Optimal Transport distances. Each color identified a set of pulses with an outliers class (-1).

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Requirements

- Deinterleaving set of pulses as an input:

Hypothesis: 1 set of pulses = 1 RADAR.

- Database with more than **60 classes** described by only three features:
 - Frequency (f_n),
 - Pulse width (pw_n),
 - Pulse Repetition of Interval (pri_n).

👉 **Identification up to the sub-mode.**

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Proximity analysis between emitters classes



Some emitters share similar characteristics and could be confused.

- **Intuition:** Compare emitters classes distributions using Optimal Transport distances.

- ✓ Green: High transport cost,
 - Emitters have different characteristics,
 - Low risk of confusion.
- ✗ Red: Low transport cost,
 - Emitters share close characteristics,
 - Easy confusion.

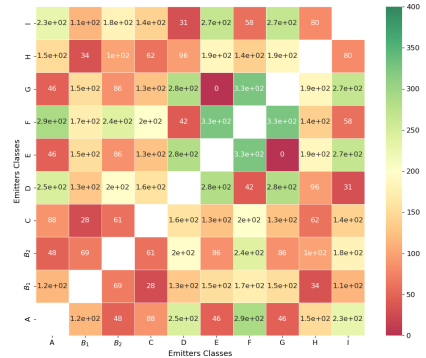


Figure: Cost matrix get from frequency and pulse width.

Optimal transport planes from frequency and pulse width

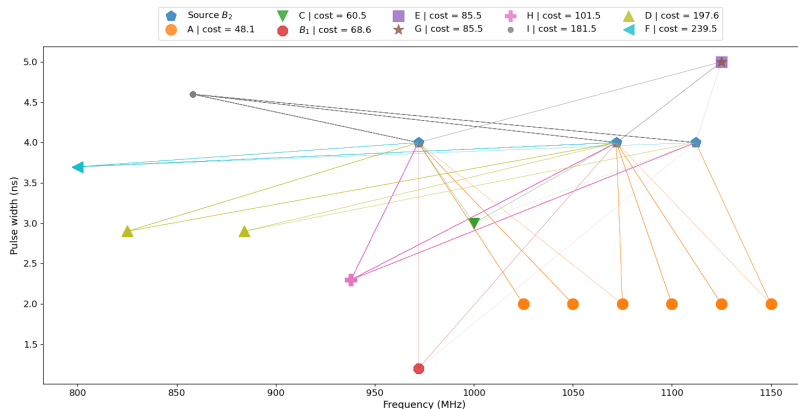


Figure: Optimal transport planes between emitter B_2 characteristics toward other emitters from Frequency and Pulse Width.

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IDentification of emitters using Optimal Transport Distances - **IDOT**

3-step strategy

- ➊ **Set of pulses representation**: Construction of a probability distribution based on the temporal emission pattern (DTOA), the frequency (F), and the pulse width (PW).
 - ➋ **Emitter class representation**: Construction of a discrete measure based on the temporal emission pattern (PRI_{mod}), the frequency (F), and the pulse width (PW).
 - ➌ **Identification of the emitter class**: Assignment of the closest class in the sense of optimal transport.
-

Step 1: Set of pulses representation

- Construction of a probability distribution from the deinterleaving set of pulses based on differentiated arrival time, frequency, and pulse width:

$$\nu = \frac{1}{M} \sum_{m=1}^M \delta_{dtoa_m, f_m, pw_m}, \quad (9)$$

with M , the number of pulses in the set and $dtoa_m = toa_m - toa_{m-1}$.

Step 2: Emitter class representation

✗ Signal can be noisy or poorly estimated:

- ⇒ Missing some pulses,
- ⇒ Distortion of the reconstruction of the PRI pattern from a set of pulses,
- ⇒ *pri* distribution of emitter from database does not correspond to the *dtoa* distribution of the set of pulses,
- ⇒ All research-based methods of PRI inefficient.

Missing pulses $\Rightarrow PRI \neq DTOA$

Simple case: Emitter with one PRI

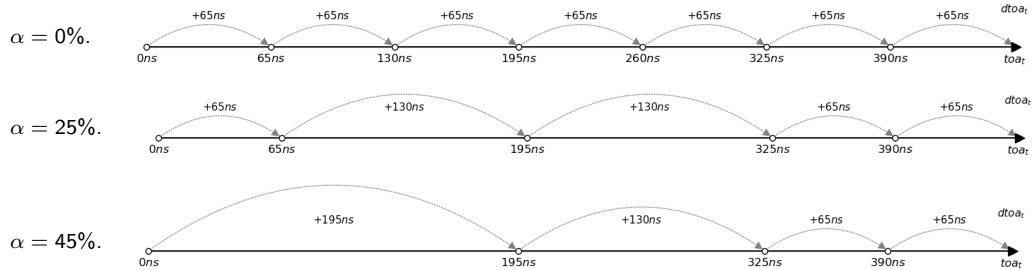


Figure: Example of the $dtoa$ values according to different missing pulse rates α , of Emitter E.



Multiple *pri* distributions.

Pulse Repetition of Interval Modeling process according to the Missing Pulses Rate

- **Intuition:** Modeling the PRI values according to the missing pulse rate,
- Using the total probability formula,
- Consider N , the discrete random variable representing the number of missing pulses.

Methodology

- I **Missing pulses representation** according to a missing pulses rate $\mathbb{P}(N = n)$,
 - II **$dtoa$ distribution representation for a fixed number of missing pulses** $\mathbb{P}(dtoa = t | N = n)$,
 - III **Representation of the $dtoa$ distribution conditionally to the number of missing pulses**, $\mathbb{P}(dtoa = t)$.
-

Focus on the discrete process

- ① Assuming that the losses are independent, a geometric distribution can represent them:

$$\mathbb{P}(N = n) = (1 - \alpha)\alpha^n, \quad (10)$$

- ② Assuming that the values of *pri* have uniform probabilities of appearance the sum of successive *pri* starting with the k^{th} value of *pri*, one have:

$$t_{nk} = \sum_{m=k}^{k+n} pri_{m, mod[K]} \Rightarrow \mathbb{P}(dtoa = t | N = n) = \frac{1}{K} \sum_{k=0}^{K-1} \delta_{t_{nk}}. \quad (11)$$

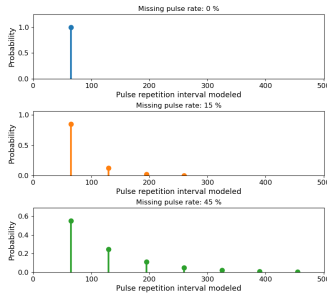
- ③ from Equations (10), and (11):

$$\nu_{\alpha} = \frac{1}{K} \sum_{n=0}^{\infty} (1 - \alpha)\alpha^n \sum_{k=1}^K \delta_{t_{nk}}. \quad (12)$$

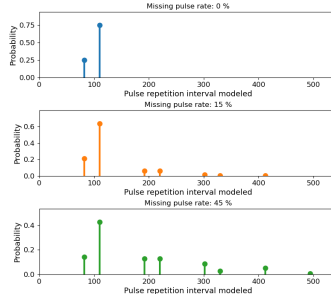
where α is the missing pulses rate, and K is the number of *pri* of the considering emitter.

- 👉 Generalizable to the continuous case by considering mixtures of Gaussians.

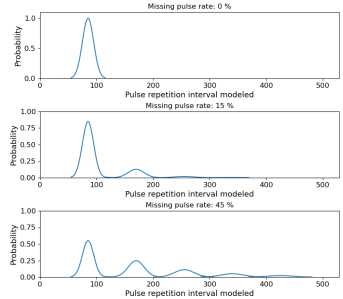
Application to Emitter E, C, and H



Emitter E with one PRI value.



Emitter C with two PRI values.



Emitter H with a random PRI.

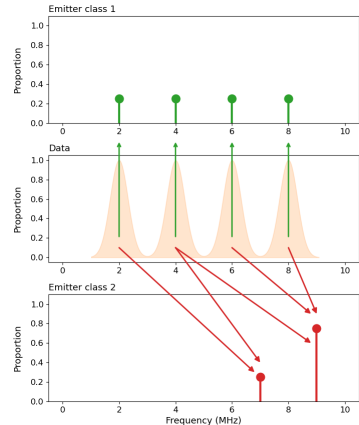
Figure: Modelization of the *doa* distributions for different emitters according to different missing pulse rates.

Step 3: Identification Algorithm

- Identification of the emitter class by assigning the closest emitter in the optimal transport distance sense to each set of pulses:

$$j^* = \operatorname{argmin}_{j \in (1, \dots, J)} d(\nu, \tau_j), \quad (13)$$

with J the number of emitters classes



Identification output for Emitter C

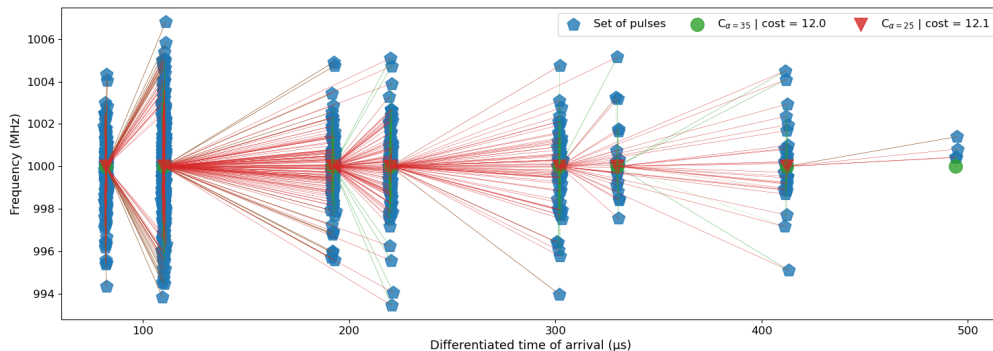


Figure: Identification outputs for Emitter C characterized by 2 PRI values ($82ns$ and $110ns$) with 35% of missing pulses. The plane overlaps the pulses and the characteristic points of the two first outputs identified by the algorithm.

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 - Evaluation of identification results
- 6 Conclusion

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 - Identification of emitters using Optimal Transport Distances
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Simulated signal

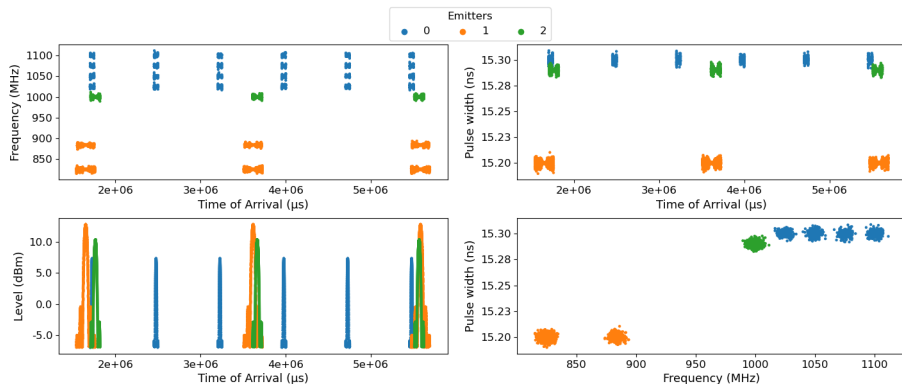


Figure: Simulated signal gathering 10044 pulses from 3 emitters, identified by a color.

Sensitivity to estimation errors

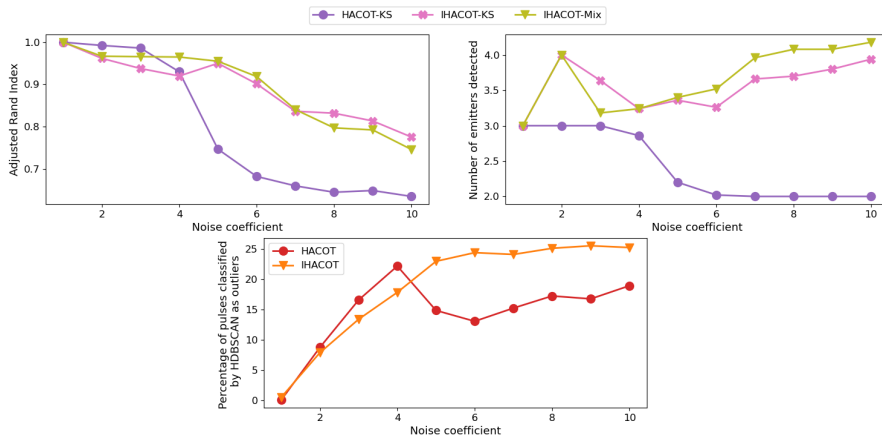


Figure: Performance of deinterleaving methods according to the noise added with ARI, number of emitters detected, and part of pulses classified by HDBSCAN as outliers. Each curve identifies a method.

Experimentation outputs

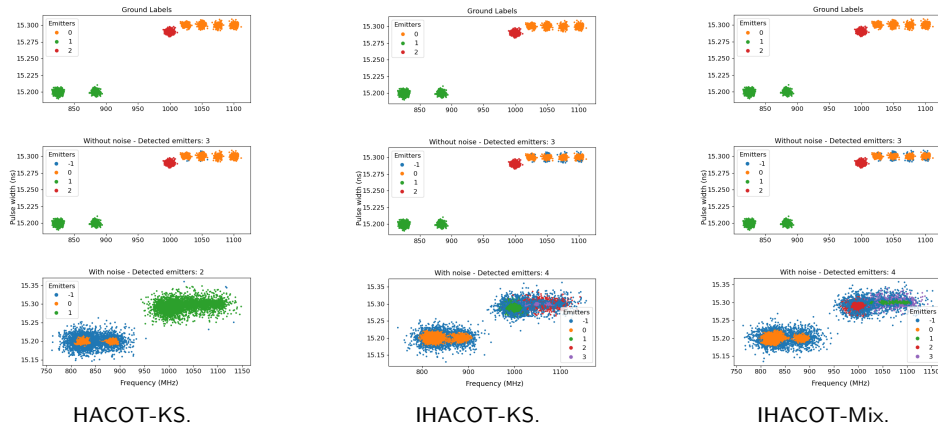


Figure: Pulse's representation in the (f_n, pw_n) plane according to the deinterleaving results without noise and the baseline level is multiplying by 4.

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Emitters classes characteristics

Name	Frequency (MHz)	Pulse width (ns)	PRI (μs)
A	1025, 1050, 1075 1100, 1125, 1150	2	1520, 1540, 1560, 1580, 1600, 1620 1640, 1660, 1680, 1700, 1720, 1740 1760, 1780, 1800, 1820, 1840, 1860 1880, 1900, 1920, 1940, 1960, 1980
B	972	1.2	2000, 2200, 2400
C	972, 1072, 1112	4	2200
D	1000	4	1700, 2200
E	825, 884	2.9	1700, 1720, 1740, 1760, 1780, 1800, 1820 1840, 1860, 1880, 2080, 2120, 2160, 2200
F	800	3.7	1700, 1800, 1900, 2000, 2100, 2200, 2300, 2400
G	938	2.3	Min: 1700, Max: 1900

Table: Simulated emitters characteristics.

Proximity between emitters

⚠ Emitters with close characteristics:

✓ High transport cost: Different characteristics

✗ Low transport cost: Risk of confusion

⚠ Less clear-cut results,

⚠ Must pay attention to all classes.

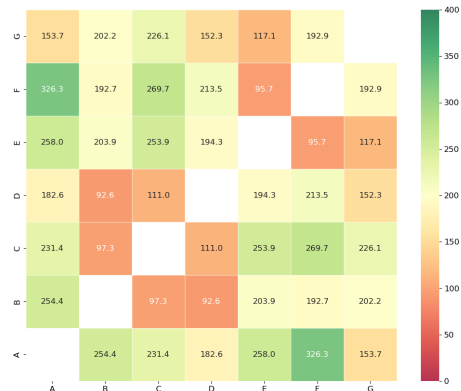


Figure: Cost matrix get from pulse repetition of intervals, frequency, and pulse width.

Robustness against mixing pulses

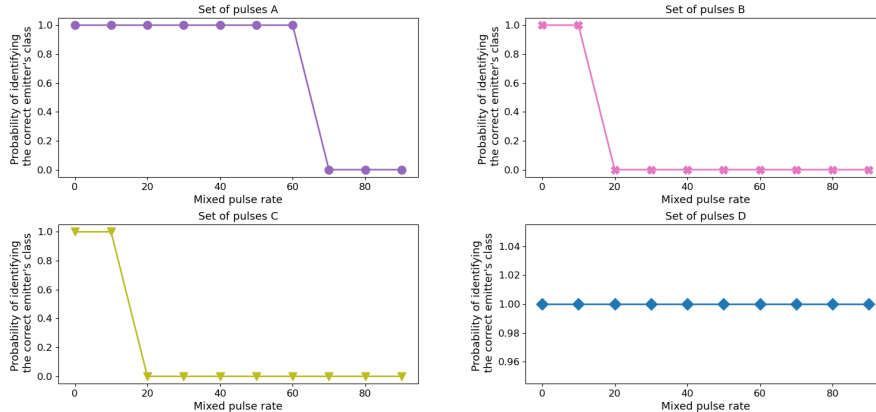


Figure: IDOT performance by analyzing the probability of identifying the correct emitter for each set of pulses according to several mixing pulse rates.

Experimentation outputs

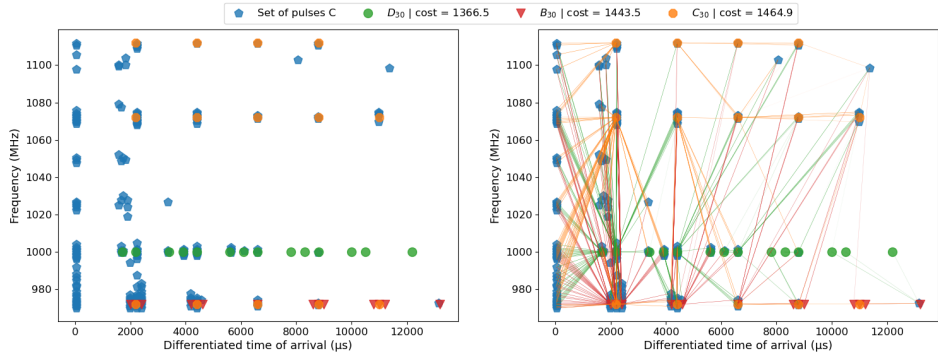


Figure: IDOT results for Set of pulses C characterized by a PRI (2200 ns) and 3 frequencies (972, 1072, 1112 MHz) with a mixing rate of 40% in the $(f_n, d\text{toa}_n)$ plane.

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Conclusions

- Development of two unsupervised deinterleaving methods based on accessible and reliable features:
 - Works on moderately complicated cases with few data,
 - Avoid merging pulses belonging to different RADARs.
- Development of a new identification method based on PRI modeling:
 - Insensitive to missing pulses,
 - Methodology can handle a large number of classes to identify.
- Empirical results highlighting the robustness of the RADAR Recognition Process.

Perspectives

- **Deinterleaving RADAR pulses:**

- Add temporarily available features and incorporate outliers in the process,
- Handle cases where the RADAR clusters are not simultaneously active,
- Analyze other breakpoints when setting the confidence threshold to stop dendrogram aggregations.

- **Identification of RADAR:**

- Explore alternative methods to represent random PRI instead of Gaussian distribution,
 - Modeling the frequency according to different levels of outliers,
 - Add temporarily available features.
- Considering adverse countermeasures introducing disruptive signals and deploying the processes in real-time.

Scientific contributions (Mottier, Chardon, and Pascal)

Articles in peer-reviewed journals

- 📄 "Specific Emitter Identification based on Optimal Transport Distances and the Missing Pulses Rate Modelization for the Pulse Repetition of Intervals." - IEEE Aerospace and Electronic Systems Society 2024 (in writing).
- ✓ "Deinterleaving RADAR emitters with Optimal Transport Distances." - IEEE Aerospace and Electronic Systems Society 2024.

Conferences with proceedings

- ✓ "Désentrelacement et classification des émetteurs RADARs basés sur l'utilisation des distances de Transport Optimal." - CAID 2022.
- ✓ "Désentrelacement et classification de signaux RADAR basés sur des distances de transport optimal." - GRETSI 2022.
- ✓ "RADAR Emitter Classification with Optimal Transport Distances." - EUSIPCO 2022.
- ✓ "Deinterleaving and Clustering unknown RADAR pulses." - IEEE Radar Conference 2021.

Patents

- ✓ "Procédé d'identification d'un émetteur radar et système d'identification associé" - 2023
- ✓ "Procédé de désentrelacement d'impulsions RADAR" - 2023



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AVANTIX


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Égalité
Fraternité*



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Complex case: Emitter with two PRI values [1/2]

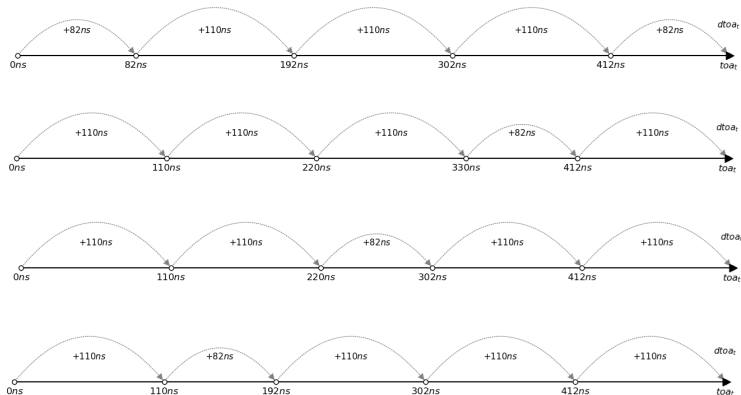


Figure: *doa* distribution when $\alpha = 0\%$ for a PRI pattern of [82, 110, 110, 110].

Complex case: Emitter with two PRI values [2/2]

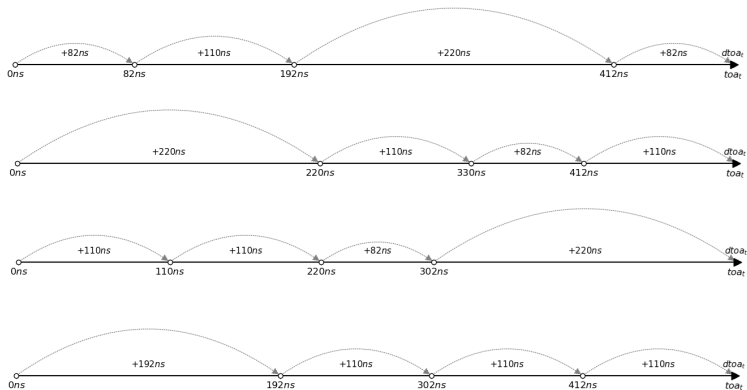


Figure: $dtot$ distribution when $\alpha = 25\%$ for a PRI pattern of $[82, 110, 110, 110]$.

Focus on the continuous process

- ① Assuming that the losses are independent, a geometric distribution can represent them:

$$\mathbb{P}(N = n) = (1 - \alpha)\alpha^n, \quad (14)$$

- ② The *pri* is represented by a Gaussian random variable J_m with mean pri_m and variance σ^2 ; assuming that the J_m are independent one have:

$$J_m \sim \mathcal{N}(pri_m, \sigma^2) \text{ and } T_{nk} = \sum_{m=k}^{k+n} J_m \text{ with } T_{nk} \sim \mathcal{N}(t_{nk}, (n+1)\sigma^2).. \quad (15)$$

- ③ from Equations (14), and (15):

$$p_\alpha(t) = \frac{1}{K} \sum_{n=0}^{\infty} (1 - \alpha)\alpha^n \sum_{k=1}^K p_{nk}(t), \quad (16)$$

where α is the missing pulses rate, and K is the number of *pri* of the considering emitter, p_{nk} is the density of the Gaussian variable T_{nk} and T_{nk} the law of the *dtoa*.

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Simulated signal

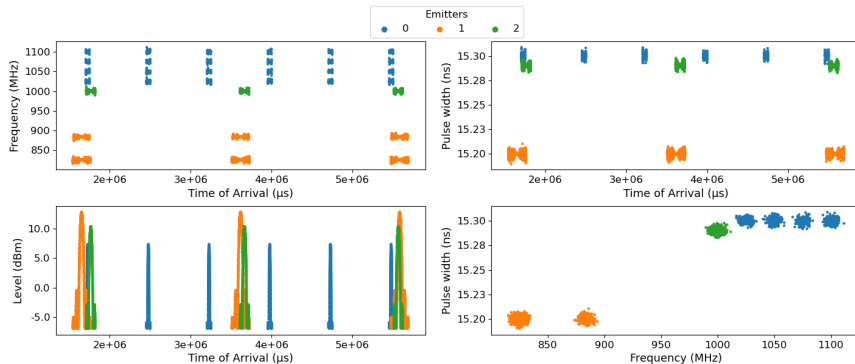
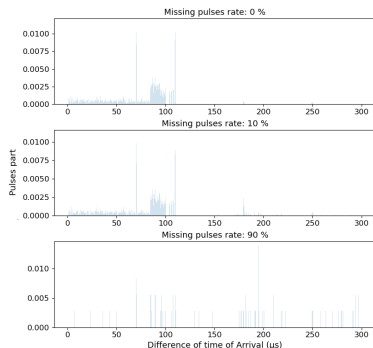
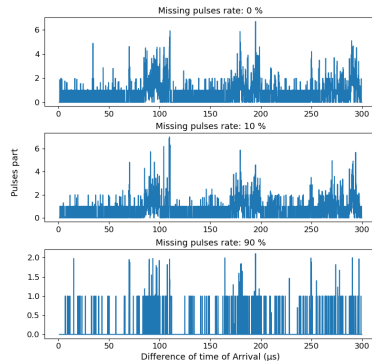


Figure: Simulated signal gathering 10044 pulses from 3 emitters, identified by a color.

Comparison with PRI-based methods



Mardia.



PRI Transform.

Figure: Outputs of the PRI-based methods to deinterleave the simulated signal according to different missing pulse rates.

Robustness to outliers

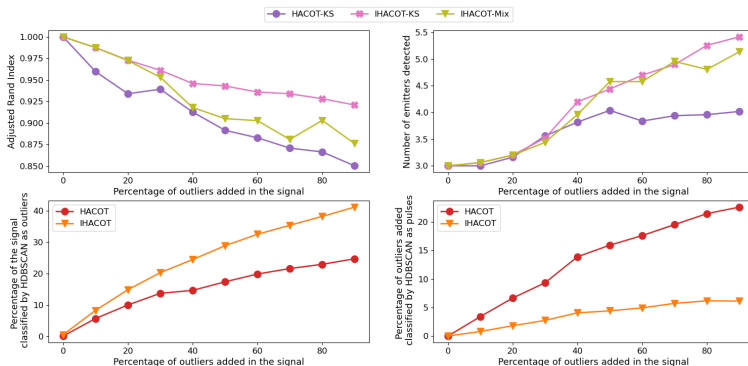
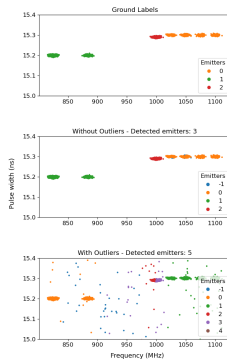
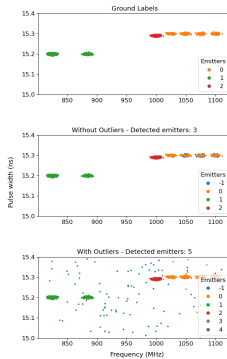


Figure: Performance of deinterleaving methods according to the outliers rate added with ARI, number of emitters detected, part of the pulses classified by HDBSCAN as outliers, and part of outliers added in the signal classified in the sets of pulses. Each curve identifies a method.

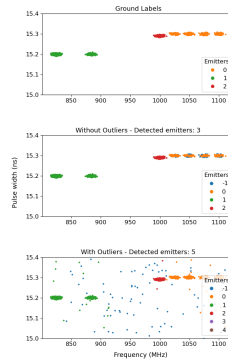
Experimentation outputs



HACOT-KS.



IHACOT-KS.



IHACOT-Mix.

Figure: Pulse's representation in the (f_n, pw_n) plane according to the deinterleaving results without outliers and when 90% of outliers is added.

Performance against missing pulses

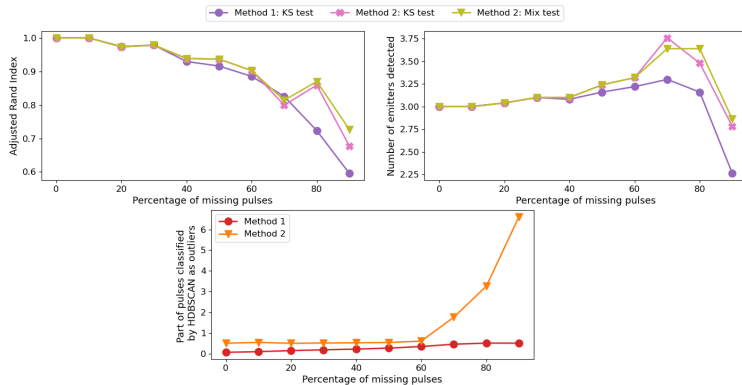
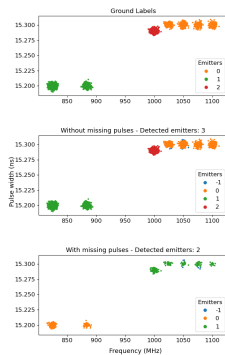
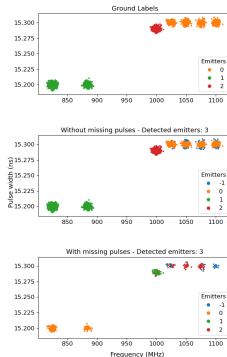


Figure: Performance of deinterleaving methods against missing pulses with ARI, number of emitters detected, and part of pulses classified by HDBSCAN as outliers. Each curve identifies a method.

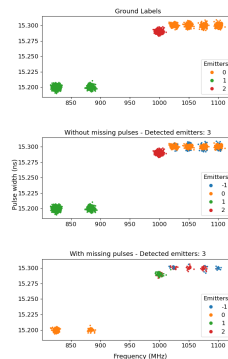
Experimentation outputs



HACOT-KS.



IHACOT-KS.



IHACOT-Mix.

Figure: Pulse's representation in the (f_n, pw_n) plane according to the deinterleaving results without missing pulses and when 90% of the pulses are missing.

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Robustness to outliers

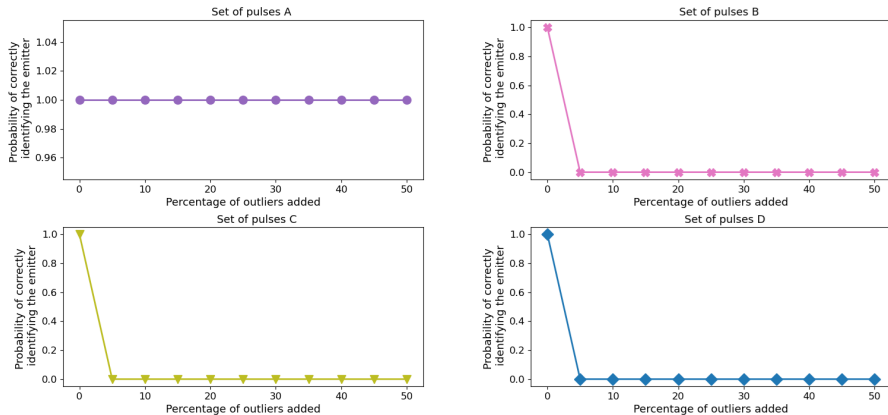


Figure: IDOT performance by analyzing the probability of identifying the correct emitter according to the outliers added for each set of pulses.

Experimentation outputs

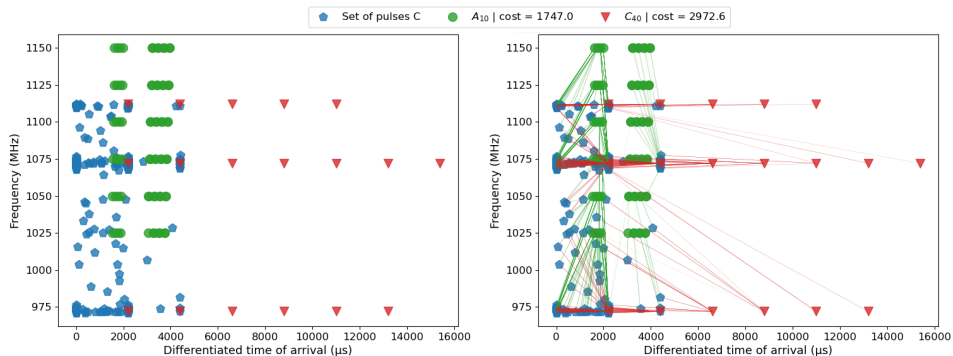


Figure: IDOT results for Set of pulses C with 40% of outliers added in the $(f_n, d\text{toa}_n)$ plane.

Sensitivity to estimation errors

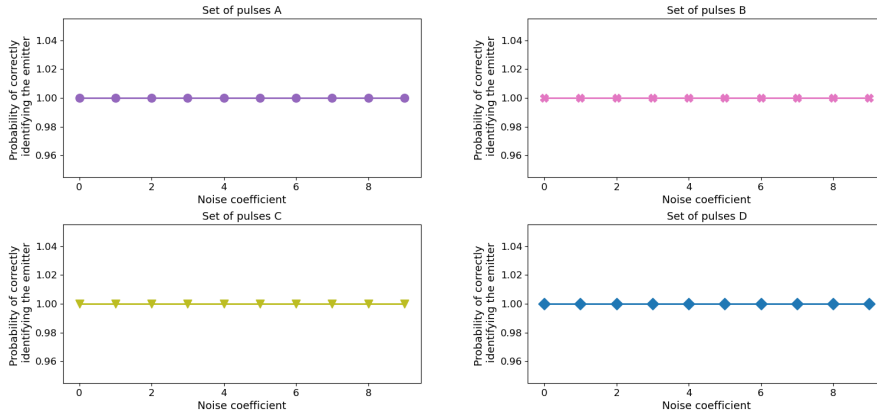


Figure: IDOT performance by analyzing the probability of identifying the correct emitter for each set of pulses when the variance level of the set of pulses is multiplied by a coefficient.

Experimentation outputs for Emitter C

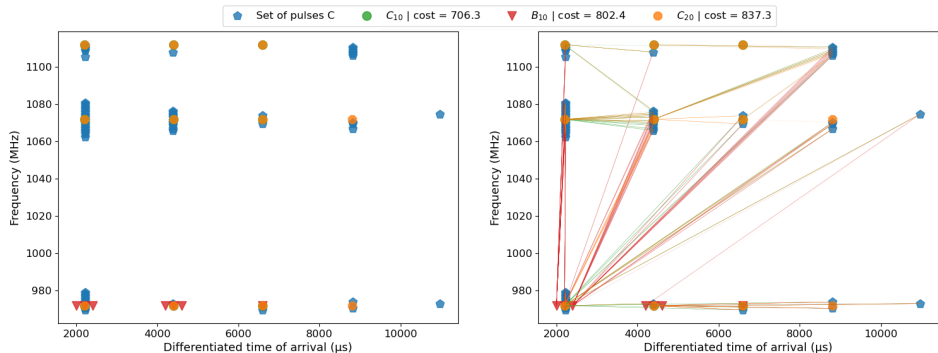


Figure: IDOT results for Set of pulses C when the baseline level is multiplying by 4.

Performance against missing pulse

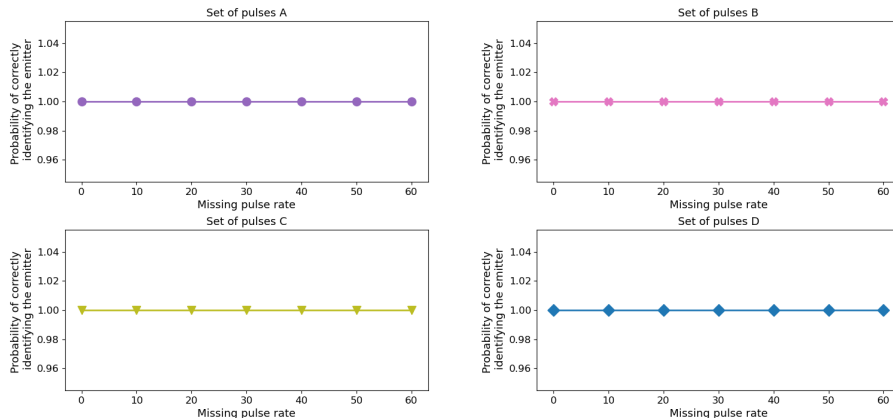


Figure: IDOT performance by analyzing the probability of identifying the correct emitter according to several missing pulse rate.

Experimentation outputs

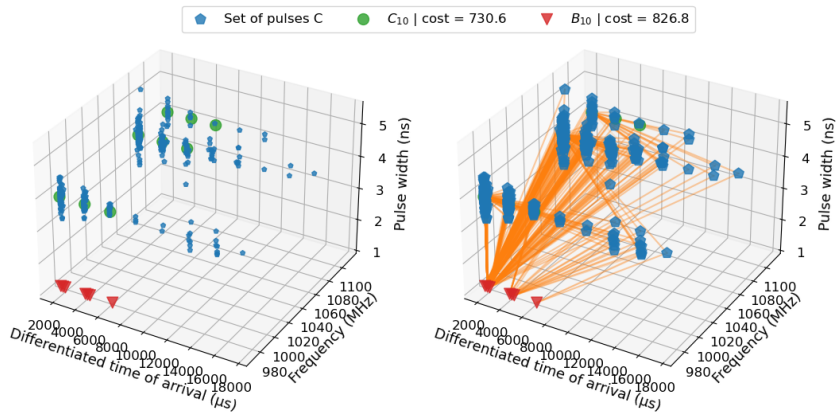


Figure: IDOT results plotted in 3 dimensions when 40% of pulses are missing for Set of pulses C.

Computational complexity

- **HACOT** [30]: $O(K^2(T^2\bar{R}^2 + \bar{C}^2B))$

with T the length of the signal, K the number of emitters, \bar{R} the mean rate of pulses, \bar{C} the mean number of clusters per emitter.

- **IHACOT**: $O(\bar{C}^3K^3T^3S^3)$

with $P = CTS$ the number of clusters identified by HDBSCAN where S is the average number of sweeps by second, and C is the number of pre-aggregation clusters.

- **IDOT** [31]: $O(B^2J)$

with J the number of classes, and B the number of bins representing the set of pulses.

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Fraternité*



Still 5 minutes to go ...

Fight against gender-based and sexual violence

En avant toute(s): the association fights for gender equality and against sexism and sexual violence. It prevents violence, supports victims via a dedicated chat, and raises awareness among young people about school equality. Its objective: a caring, egalitarian society without discrimination or violence. [\[website\]](#)



Fondation des femmes: the Women's Foundation is the reference structure in France for the freedom and rights of women and against the violence of which they are victims. Supported by donations, this organization offers financial, legal, and material assistance to impact community initiatives across the country. [\[website\]](#)



Çapèse: it aims to promote gender equity within the CentraleSupélec campus. [\[website\]](#)



Fight against PhD students' harassment/discrimination

- **No association** specializing in the fight against discrimination and harassment of doctoral students...
- Some contacts:
 - Your PhD student representatives...
 - **CentraleSupélec**: work psychologist, official report ... [\[resources\]](#)
 - **Paris-Saclay University**: official report [\[ressources\]](#)
 - **JuriSup**: a professional network of legal affairs managers in higher education and research [\[ressources\]](#)
 - **Legal support**

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